



## A Whole Different Serving of Tapioca: Exploring Just Intonation Dissonance

by Kyle Gann

Just Intonation dissonance, said Harry Partch in one of tuning theory's more ringing phrases, is "a whole different serving of tapioca" from equally tempered dissonance. I had no idea what that meant when I first heard it. I've now used enough Just Intonation dissonance in my music to learn how right he was.

My early Just Intonation works were rather fanatically consonant. Having expended years of time and energy to achieve pure consonance, I was at a loss as to how to justify returning to dissonance, and as to what strategy or theory to use as a guideline. After all, if you start working in Just Intonation to get rid of beats in your sonic surface, under what theoretical aegis do you bring beats back in? And so I was in danger of replicating the fate of one of our American forebears, William Billings (1746–1800), who was criticized because his hymns made almost no use of dissonance. And for exactly parallel reasons: Billings had no theoretical sanction for breaking the rules that he had learned at such cost. In response, Billings

wrote a song, "Jargon," completely in seconds, sevenths, and ninths, and justified them with a satirical text ("Let horrid Jargon split the air/And rive the Nerves asunder"). Somewhat analogously, I found a text and allowed programmatic considerations to lead me into several different kinds and degrees of just dissonance.

My text was taken from *My Life on the Plains*, the autobiographical memoir of General George Armstrong Custer.<sup>1</sup> If ever a text cried out for dissonance, it was this, bristling with unacknowledged contradictions. One moment, Custer would admit that if he were an Indian, he would ignore U.S. Government mandates and go live on the plains; a few pages later he would damn all Indians as brutes only governable by force.

Overall, the book seemed schizophrenic, so for "Custer: 'If I Were an Indian...'"—the first movement of my music-theater piece *Custer and Sitting Bull*<sup>2</sup>—I came up with a schizophrenic scale. Half of the scale was a normal major/minor Just Intonation scale on C (Table 1a).

(text continued on page 4)

Table 1a										
Pitch	C	D	E <sup>b</sup>	E	F	G	A <sup>b</sup>	A	B <sup>b</sup>	B
Ratio	1/1	9/8	6/5	5/4	4/3	3/2	8/5	5/3	9/5	15/8
Cents	0	203.9	315.6	386.3	498.0	702.0	813.7	884.4	1017.6	1088.3
Table 1b										
Pitch	C <sup>1+</sup>	C <sup>♯1+</sup>	D <sup>1</sup>	E <sup>1+</sup>	F <sup>1+</sup>	F <sup>♯1+</sup>	G <sup>1</sup>	A <sup>1+</sup>	B <sup>b1</sup>	B <sup>1</sup>
Ratio	2673/2560	4455/4096	297/256	2673/2048	891/640	1485/1024	99/64	891/512	297/160	495/256
Cents	74.8	145.5	257.2	461.1	572.8	643.5	755.2	959.1	1070.9	1141.5

Example 1

Musical score for Example 1, featuring Voice, English Horn, Marimba, Drum, and Bass. The score is in 4/4 time and includes lyrics: "In - di - an can ne - ver be per - mit - ted to view the ques - tion in this de - li - be - rate way. When the".

(*Tapioca... continued from page 1*)

The other half was this same scale transposed upward 257.2 cents to the 297 harmonic: that is, as a quick calculation will show, to the eleventh harmonic of the Pythagorean major sixth. Stated in terms of C, the other scale's numbers looked more impressive (given in Table 1b in the specific Just Intonation pitch notation devised by my teacher Ben Johnston).

Keep in mind that although I have listed this second scale starting from C<sup>†</sup> so it can be seen in conjunction with the just scale on C, its actual tonic is D<sup>†</sup>. Recalculate the ratios with <sup>297</sup>/<sub>256</sub> as <sup>1</sup>/<sub>1</sub>, and you'll find the same ratios as in the C major/minor scale in Table 1a.

Thus, the entire movement operated within a bitonal, twenty-pitch just pitch collection, with one tonic on C and the other on D<sup>†</sup>, the 297th harmonic of C. I chose D<sup>†</sup> explicitly for its approximation of quarter-tone values. Although I would never compose in equally tempered quarter tones, considering them merely a convenient addition to an equally tempered twelve-pitch scale that was a historical mistake in the first place, I admit I've always found quarter tones rather raucously titillating to the ear, and I wanted to build in as much incommensurability to Custer's scale as possible. Also, I've come to

think of eleven as the number of mediation, providing as it does so many midpoints between close 5- and 7-limit intervals, so 11 times 27 promised to mediate among my other pitches.

Among the paragraphs I chose from Custer's book were evident many different degrees and levels of hypocrisy, a real connoisseur's cornucopia of mendacity and rationalization. At times, Custer was capable of very reasonably portraying the Indian people as victims of white imperialist aggression. At these moments, I had the music move fluidly back and forth between the two scales, as in Example 1. When he began justifying the Washita massacre, however, I used intrusions in F<sup>†</sup> to interrupt the E<sup>b</sup> major statement of "Garry Owen," the 7th Cavalry's theme song, as in Example 2. When Custer reached the height of hypocrisy, accusing the Indians of depredations that were actually the direct result of white actions, I undercut his sanctimonious speech in E<sup>b</sup> major with a bass line and a soprano countermelody in the D<sup>†</sup> scale as in Example 3. And finally, after his plea for mercy at his 1868 court martial ("Judge me not by what is known now, but in the light of what I knew when these events transpired"), I climaxed with a final chord in both keys at once (Example 4).

Example 2

Musical score for Example 2, featuring Voice, Flute, English Horn, and Brass. The score is in 2/8 time and includes lyrics: "My firm con - vic - tion based on a - na - ly - sis of the cha - rac - ter traits of the In - di - an".

## Example 3

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Voice: Al- though ne- ver claimed as an ex- po- nent of the peace po- li- cy, Yet I en- ter- tained the most peace- a- ble sen- ti- ments

Flute

Trumpet in B $\flat$

Bassoon

Double Bass

I felt that these varied deployments of bitonal just scales—successive, simultaneous, layered, fused—offered me a nuanced way to comment on Custer’s words, as though the C scale were his conscious mind and the D $\sharp$  scale his unconscious poking its head out at inopportune (for Custer) moments. Hypocrisy, however, is a rather superficial emotional state, and this was still a superficial approach to Just Intonation dissonance. Similar, if less subtle, effects could have been achieved in equal temperament. The third movement of *Custer and Sitting Bull* demanded a deeper treatment. This was “Sun Dance/Battle of the Greasy Grass River,” a depiction of the sun dance that Sitting Bull led before the battle of the Little Bighorn (the Sioux name for the river translates as “Greasy Grass River”), along with the subsequent battle. During the sun dance, Sitting Bull cut a hundred notches of flesh out of his arms and legs, letting the blood run down until he had a vision. This sounded uncomfortable. Dissonance seemed appropriate.

To symbolize Sitting Bull’s trance and vision, I wanted intervals that would pulsate with acoustic beats. In particular, I was curious to work with  $^{40}/_{27}$ , the notorious “wolf fifth” that caused so many sixteenth-

century headaches because, occurring between D and A in a justly tuned scale based on C, it made the ii chord unavailable in a twelve-pitch just tuning. Besides, it seemed appropriate that a Native American sun dance would be pervaded by the spirit of the wolf.

I set up a series of drones (Table 2) that would move gradually from the interval of a fourth to one of a fifth, with  $^{40}/_{27}$  as the penultimate interval before reaching

Pitch	Ratio	Cents
B $\flat$	$^{21}/_{16}$	470.8
B-	$^4/_3$	498.0
B	$^{27}/_{20}$	519.6
B $\sharp$ -	$^{11}/_8$	551.3
C $\flat$	$^7/_5$	582.5
B $\sharp$ -	$^{10}/_7$	617.5
C $\sharp$	$^{16}/_{11}$	648.7
C $\sharp$ -	$^{40}/_{27}$	680.5
C $\sharp$	$^3/_2$	702.0

## Example 4

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Voice: Ask the Sax - on race! Ask the Sax - on race!

Marimba

Winds

the consonant goal  $3/2$ . I began, though, not with  $4/3$ , but  $21/16$ , an interval I had come to enjoy from La Monte Young's use of it in *The Well-Tuned Piano*. Over a tonic  $F\sharp$ , the series of nine drone intervals turned out as follows (pitches once again with Johnston's accidentals).

This list includes four tritones, two septimal (seven-based) and two undecimal (eleven-based). Not included, interestingly (to me), was  $45/32$ , the somewhat historically standard European tritone of 590 cents. Only eight cents away,  $7/5$  was so close as to render it superfluous. Then also, it is a more complex ratio than any of the tritones I already have.

My approach to tuning has generally aimed at rendering complex intervals as intelligible as possible, and I continued that strategy with this series. Each interval implied a certain harmony and tonality according to where it fell in a harmonic series.

$B7^{21/16}$ , the septimal fourth, is, of course, the seventh harmonic of a  $3/2$ . With reference to the tonic  $F\sharp$ , the pitch implied a dominant seventh on  $C\sharp^{3/2}$ .

$B-^{4/3}$  is the traditional subdominant. Its consonance with the tonic was unproblematic, and many pitches could be used with both for a fair amount of consonance.

$B^{27/20}$  was more interesting. Harmonics 27 and 20 imply an interval between the Pythagorean major sixth ( $27/16$ ) and a five-limit major third ( $5/4$ , or  $20/16$ ). Therefore  $D-^{8/5}$  was an important note to include, as being a fundamental of which  $F\sharp^{1/1}$  and  $B^{27/20}$  are both harmonics.

$B\uparrow-^{11/8}$  was again a simpler case, since it is the eleventh harmonic of  $F\sharp$ . I harmonized it with other low-numbered overtones of  $F\sharp$ .

$C7^{7/5}$  and  $B\sharp-^{10/7}$ , the midpoints between the perfect fourth and perfect fifth, are an interesting pair. The numbers suggest that  $C7$  is the seventh harmonic in a harmonic series of which  $F\sharp$  is the fifth harmonic; that harmonic series has  $D-^{8/5}$  as its fundamental. Likewise,  $10/7$  suggests that  $B\sharp-$  is the tenth (or fifth) harmonic in a series in which  $F\sharp$  is the seventh; that series is founded on  $G\sharp-^{8/7}$ . Thus, every time in the music that the drone pitch shifted from  $C7^{7/5}$  to  $B\sharp-$  (only 35 cents, after all) the tonality tended to flip-flop a tritone, from  $D-$  to  $G\sharp-$  or back. Both in classical music, where they represent Neapolitan or German sixth progressions, and in jazz where the tritone substitution chord is involved, tritone root movements between seventh chords tend to guarantee interesting and effective voice leading. Having that  $7/5-10/7$  progression right in the middle was a juicy

contrast to work with.

$C\sharp^{16/11}$  is the eleventh undertone; in other words, the pitch of which  $1/1$  is the eleventh harmonic. I harmonized it with its third harmonic,  $12/11$ , and also with  $4/3$ , which made a  $12/11$  interval with it.

$C\sharp-^{40/27}$  was the crucial interval, the wolf, the penultimate pitch before the climactic resolution into  $C\sharp^{3/2}$ . The ratio implies that the pitches involved,  $F\sharp$  and  $C\sharp-$ , are the second and sixth scale degrees of a scale on  $E-^{16/9}$ . Thus, I used  $16/9$ , and also the major third of  $16/9$ ,  $10/9$ , as mediating pitches.

$C\sharp^{3/2}$  was the sun dance's consonant goal, coinciding with Sitting Bull's vision: white soldiers and cavalry falling down, as a voice said, "I give you these because they have no ears." This drone I was able to harmonize with all of the small-number ratios I had available.

This information is summarized in Example 5, which shows the drone interval, the root of the harmonic series implied, and the scales I spun my melodies around to make each drone interval intelligible.

An even more dissonant passage than the sun dance was required by the aftermath of the battle scene. Here, as the 200-plus dead soldiers lay on the battlefield beginning to swell and stink, I wanted no mediating consonances, just an unintelligible sonority to match the unfathomability of the scene.

In the bass I used the drones that had run throughout the sun dance. Above those, quite intuitively, I chose pitches from the ones I had available that I suspected would be particularly incommensurate. Specifically, for a drone whose ratio had a 7 in the numerator, I would pick a pitch with a 7 in the denominator, for example,  $8/7$  above  $21/16$  and again above  $7/5$ . The seven chords in the battle aftermath are shown in Table 3.

The most complex of these are the chords that last

$7/4$	$15/8$	$25/14$	$9/5$	$45/28$	$7/4$	$25/14$
$12/11$	$8/7$	$8/7$	$7/6$	$10/9$	$12/11$	$27/20$
$1/1$	$1/1$	$1/1$	$1/1$	$1/1$	$1/1$	$1/1$
$27/20$	$21/16$	$7/5$	$11/8$	$16/11$	$10/7$	$40/27$
<b>Cents above <math>F\sharp</math></b>						
968.8	1088.3	1003.8	1017.6	821.4	968.8	1003.8
150.6	231.2	231.2	266.9	182.4	150.6	519.6
0	0	0	0	0	0	0
519.6	470.8	582.5	551.3	648.7	617.5	680.5

### Example 5

Example 5 consists of two systems of musical notation, each with a treble and bass staff. The first system has four measures. Above the notes are ratios:  $\frac{1}{1}$ ,  $\frac{7}{6}$ ,  $\frac{21}{16}$ ,  $\frac{3}{2}$ ,  $\frac{7}{4}$ ,  $\frac{1}{1}$ ,  $\frac{10}{9}$ ,  $\frac{4}{3}$ ,  $\frac{3}{2}$ ,  $\frac{16}{9}$ ,  $\frac{9}{5}$ ,  $\frac{27}{20}$ ,  $\frac{6}{5}$ ,  $\frac{9}{8}$ ,  $\frac{1}{1}$ ,  $\frac{1}{1}$ ,  $\frac{9}{8}$ ,  $\frac{11}{8}$ ,  $\frac{3}{2}$ ,  $\frac{1}{1}$ ,  $\frac{6}{5}$ ,  $\frac{7}{5}$ ,  $\frac{8}{5}$ ,  $\frac{9}{5}$ . Below the notes are ratios and cents:  $\frac{21}{16}$  470.8,  $\frac{4}{3}$  498.0,  $\frac{27}{20}$  519.6,  $\frac{11}{8}$  551.3,  $\frac{7}{5}$  582.5. The second system has four measures. Above the notes are ratios:  $\frac{1}{1}$ ,  $\frac{8}{7}$ ,  $\frac{10}{7}$ ,  $\frac{12}{7}$ ,  $\frac{1}{1}$ ,  $\frac{12}{11}$ ,  $\frac{4}{3}$ ,  $\frac{12}{11}$ ,  $\frac{1}{1}$ ,  $\frac{10}{9}$ ,  $\frac{4}{3}$ ,  $\frac{40}{27}$ ,  $\frac{16}{9}$ ,  $\frac{1}{1}$ ,  $\frac{9}{8}$ ,  $\frac{7}{6}$ ,  $\frac{6}{5}$ ,  $\frac{5}{4}$ ,  $\frac{4}{3}$ ,  $\frac{3}{2}$ ,  $\frac{8}{5}$ ,  $\frac{5}{3}$ ,  $\frac{7}{4}$ . Below the notes are ratios and cents:  $\frac{10}{7}$  617.5,  $\frac{16}{11}$  648.7,  $\frac{40}{27}$  680.5,  $\frac{3}{2}$  702.0. Implied and Fundamental ratios are listed below the first system.

Implied				
Fundamental:	1/1 0.0	4/3 498.0	8/5 813.7	1/1 0.0
				8/5 813.7

the longest, the third from last—at ratios of 1386:3080:4032:4455—and the last, at ratios of 1890:5103:5600:6750. The chords themselves contain no close pitches or tiny intervals; in spacing, they approximate the seventh chords of tonal music, revolving roughly around a half-diminished seventh chord, spelled upward as C#, F#, A, D#. The half-diminished seventh chord is a traditional device for creating musical tension. Tuned in incommensurate just intervals, it takes on a whole new aura of dissonance.

With a good sound system, and played at high volume, these seven chords growl and pulsate and throb in unpredictable acoustic patterns, presenting just the image I wanted: the stillness of death (since the fundamentals of these pitches remain motionless), but seething with emotion and ominous implications.

Looking at the waveforms of these chords on a sine wave chart or oscilloscope, their difference from equal tempered dissonance is visible. Equally tempered chords

will mimic Just Intonation chords at first, looking initially regular in their patterns, but then flattening out as their ratios go out of sync. These just chords, however, look weirder as they begin, but one can find large-scale periodicities recurring at different rates, for a truly multidimensional sonority.

I am a long way from coming to any general theory of Just Intonation dissonance. But I am already impressed at how vastly expanded the range of consonance and dissonance is in Just Intonation, from beatless note fusions that blend into a single timbre to wild assemblages that beat on several different levels at once. It's great news for someone who can't stand serving an audience the same old tapioca they've had before.

#### Notes:

1. University of Oklahoma Press. Norman, OK: 1962.
2. On the CD *Custer's Ghost*. Monroe Street MSM60104. Available from the Just Intonation Store.